Math 6 Unit 3: Expressions

4 weeks (Math 67 Unit 3: 3 weeks)

In this unit students will:

- Represent repeated multiplication with exponents.
- Evaluate expressions containing exponents to solve mathematical and real world problems.
- Translate verbal phrases and situations into algebraic expressions.
- Identify the parts of a given expression.
- Use the properties to identify equivalent expressions.
- Use the properties and mathematical models to generate equivalent expressions.

Unit Resources:

- Unit 3 Overview Video
- Student Friendly Standards
- Parent Letter
- Vocabulary Cards
- Sample Concept Map
- Prerequisite Skills Assessment
- Sample Post Assessment
- Culminating Task

Topic 1: Writing and Evaluating Expressions

Big Ideas/Enduring Understandings:

- Variables can be used as unique unknown values or as quantities that vary.
- Exponential notation is a way to express repeated products of the same number.
- Algebraic expressions may be used to represent and generalize mathematical problems and real life situations.
- Properties of numbers can be used to simplify and evaluate expressions.

Essential Questions:

- How are “standard form” and “exponential form” related?
- What is the purpose of an exponent?
- How are exponents used when evaluating expressions?
- How is the order of operations used to evaluate expressions?
- How are exponents useful in solving mathematical and real world problems?
- How are properties of numbers helpful in evaluating expressions?
- What strategies can I use to help me understand and represent real situations using algebraic expressions?

Student Relevance:

- Simplify lists

- Collecting data
- Summarizing

Content Standards

Content standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

MGSE6.EE.1 (600Q) Write and evaluate expressions involving whole-number exponents.

MGSE6.EE.2 (750Q) Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a (750Q) Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5-y.*

MGSE6.EE.2b (800Q) Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

MGSE6.EE.2c (840Q) Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

Vertical Alignment

5th Grade Standards	**7th Grade Standards**
MGSE5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **MGSE5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them	**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example a + 0.05a = 1.05a means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

Instructional Strategies

EE.1

Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction.

For example: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ equals $\frac{1}{16}$

Students recognize that an expression with a variable represents the same mathematics (i.e. x^4 can be written as $x \cdot x \cdot x \cdot$) and write algebraic expressions from verbal expressions.

Write the following as a numerical expressions using exponential notation.

- The area of a square with a side length of 8 m. (*Solution*: 82 m^2)
- The volume of a cube with a side length of 5 ft. (*Solution*: 53 ft^3)
- Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own. (*Solution*: 23 mice)

Evaluate:

- 4^3 (*Solution*: 64)
- $5 + 2^4 \cdot 6$ (*Solution*: 101)

Students should now focus on what terms are to be solved first **rather than** invoking the PEMDAS rule. Likewise, the division symbol (3 ÷ 5) was used and should now be replaced with a fraction bar ($\frac{3}{5}$). Less confusion will occur as students write algebraic expressions and equations if *x* represents only variables and not multiplication. The use of a dot (•) or parentheses between numbers is preferred. Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression $x - 10$ could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees," "I scored ten fewer points than my brother," etc. Students should also read an algebraic expression and write a statement. Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression $x + x + x + x + 4 \cdot 2$, students could write $2x + 2x + 8$ or some other equivalent expression. Make the connection to the simplest form of this expression as $4x + 8$. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, unifix cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical.

Provide expressions and formulas to students, along with values for the variables so students can evaluate the expression. Evaluate expressions using the order of operations with and without parentheses.

Include whole- number exponents, fractions, decimals, etc. Provide a model that shows step-by-step thinking when simplifying an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in Grade 6 The Number System; students are developing the concept and not generalizing operation rules.

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow you to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2, $5x$, xy, and $2(x + 5)$.

EE.2

Students write expressions from verbal descriptions using letters and numbers. Students understand order is important in writing subtraction and division problems. Students understand that the expression "5 times any number *n*" could be represented with $5n$ and that a number and letter written together means to multiply.

Students use appropriate mathematical language to write verbal expressions from algebraic expressions. Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms.

Students evaluate algebraic expressions, using order of operations as needed. Given an expression such as $3x + 2y$, find the value of the expression when *x* is equal to 4 and *y* is equal to 2.4. This problem requires students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate.

$$3 \cdot 4 + 2 \cdot 2.4$$

12 + 4.8

16.8

Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. For example, it costs $100 to rent the skating rink plus $5 per person. The cost for any number (n) of people could be found by the expression, $100 + 5n$. What is the cost for 25 people?

Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions.

Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes.

Consider the following expression: $x^2 + 5y + 3x + 6$ The variables are x and y.

- There are 4 terms, x^2, $5y$, $3x$, and 6.
- There are 3 variable terms, x^2, $5y$, $3x$. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term 5y represent 5 y's or $5 \cdot y$.
- There is one constant term, 6.
- The expression shows a sum of all four terms.

Examples:

- 7 more than 3 times a number (*Solution*: $3x + 7$)
- 3 times the sum of a number and 5 (*Solution*: $3(x + 5)$
- 7 less than the product of 2 and a number (*Solution*: $2x - 7$)
- Twice the difference between a number and 5 (*Solution:* $2(z - 5)$)
- Evaluate $5(n + 3) - 7n$, when $n = \frac{1}{2}$.
- The expression c + 0.07c can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25.

The perimeter of a parallelogram is found using the formula $p = 2l + 2w$. What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches.

- It is expected that students will have prior knowledge/experience related to using parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols, writing and interpreting numerical expressions, generating two numerical patterns using two given rules, interpreting a fraction as division, and operations with whole numbers, fractions, and decimals. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these concepts.
- The Conceptual Framework may be used to build teacher understanding of modeling expressions.

- Sample Lesson Plan
  - Exponents Lesson (PowerPoint)

Opening

- Verbal Phrase Organizer – Graphic Organizer for mathematical operations synonyms
  - Students will work in pairs or the teacher could lead as a whole-group discussion to complete the organizer.
- Symbols Tree Map (Answer Key) – Graphic Organizer for mathematical operations synonyms w/symbols
- Hopscotch Order of Operations or Order of Operations Graphic Organizer
- **Caution** - PEMDAS (parentheses, exponents, multiply, divide, add, subtract) is a common mnemonic device that can lead to some confusion. Because multiply and divide are listed in a hierarchal order, it is easy for students to assume that multiplication is first. However, multiplication or division is implemented based on which operation comes first from left to right.
- Study Jams - Order of Operations
- PEMDAS with Aunt Sally Smart Board Activity
- Combining Like Terms Presentation
- **Hot Seat** - Prior to the beginning of class, the teacher will prepare four or five writing or evaluating expressions questions and write them on sticky notes. Sticky notes are placed underneath student desks/chairs so that they are hidden from view. At the start of the class, inform students that several of them are sitting on "Hot Seats" and will be asked to answer questions related to the topic of study for the day. Students who have questions on sticky notes will then take turns reading the question and attempting to provide an answer.

Work Session

- Students should think of variables as quantities that vary instead of as letters that represent set values. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.
  - t + 13 as "some number plus 13 as well as "t plus 13"
  - h • 8 as "some number times 8 as well as "h times 8"
  - $\frac{k}{5}$ and k ÷ 5 as "as some number divided by 5" as well as "k divided by 5"
- Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions.
- Writing Expressions for Word Phrases – Smart Board Activity
- Coming to Terms – The teacher reads expressions aloud, and the students build expressions in pairs or small groups. You will need to create cards with appropriate symbols and numbers.
- Writing Algebraic Expressions – Students could complete the activity individually and pair with another student to compare responses.
- Algeblocks Introduction for Students (Handout)
- Combining Like Terms Using Algeblocks (Handout)
- **Performance Task Options**:
  - Exponents – In pairs, students will complete the exploration activity. You will need two different colored die (one for base, one for exponent).
  - Rules for Exponents – In pairs, students will determine if the expressions are equivalent. Students could complete the Justify Your Answer Organizer (2 per page) when solving the expressions.

- Writing and Evaluating Expressions (One Scaffolding questions and Questions that have multiple parts)
- From Situations to Expressions and Answer Key (5 situation questions)
- Algebraic Expression Card Game (2 – 4 players – matching card game)

Closing

- Algebraic Expressions Matching (3 per page)
- Number Cube Game – In pairs, students will use their knowledge of order of operations to get a specific number by rolling 4 dice.
- Order of Operations Exit Ticket (3 questions)
- **Quick Wipe Off Board Check** – Project the problems on the board and allow students to answer on the wipe off boards.
  - Mrs. Clody is selling tickets for the school dance. Each ticket costs $6. Write an algebraic expression that shows the total money made by selling *t* tickets.
  - Your grade level is going to the High Museum of Art for a field trip. Since there will be both students and adults going on this trip, Mr. Dean looks up ticket prices for both students and adults. He discovered that adult tickets cost $35 each and student tickets cost $25 each. Write an expression to represent the total cost for your grade to go on the field trip. If 193 students and 20 adults go on the field trip, what will be the total cost for the tickets purchased?
  - Tickets to a science museum cost $19.95 each. There is a $3 charge for each order no matter how many tickets are ordered. Write an expression for the cost (in dollars) of ordering *t* tickets. Then find the total cost if you order five tickets.

Exemplars for Constructed Response Prompts:
EE.2c Variable Dilemma

Engage NY Lessons:
Grade 6 Module 4: Expressions and Equations

Topic A: Relationships of the Operations

- Lesson 1: The Relationship of Addition and Subtraction
- Lesson 2: The Relationship of Multiplication and Division
- Lesson 3: The Relationship of Multiplication and Addition
- Lesson 4: The Relationship of Division and Subtraction

Topic B: Special Notations of Operations

- Lesson 5: Exponents
- Lesson 6: The Order of Operations

Topic C: Replacing Letters and Numbers

- Lesson 7: Replacing Letters with Numbers

- Lesson 8: Replacing Numbers with Letters

Topic E: Expressing Operations in Algebraic Form

- Lesson 15: Read Expressions in Which Letters Stand for Numbers
- Lessons 16–17: Write Expressions in Which Letters Stand for Numbers

Differentiation

Make use of algeblocks or algebra tiles for conceptual understanding of 2x and x^2 for struggling learners.

Common Misconceptions

- Misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like, x^3, $4x$, $3(x + 2y)$ is critical. The fact that x^3 means $x \cdot x \cdot x$; x times x times x, *not 3 times x*; $4x$ means 4 times x or $x+x+x+x$.
- The mnemonic PEMDAS can mislead students into thinking that addition must come before subtraction and multiplication must come before division.
- Students fail to see juxtaposition (side by side) as indicating multiplication. For example, evaluating 3x as 35 when x = 5 instead of 3 times 5 = 15. Also, students may rewrite 8 – 2a as 6a.
- Students also miss the understood "1" in front of a lone variable like a or x or p. For example, not realizing that 4a + a is 5a.

Evidence of Learning

By completion of this lesson, students should be able to:

- Write and evaluate numerical expressions involving whole number exponents
- Write, read, and evaluate expressions in which letters stand for numbers
- Identify parts of an expression using mathematical terms
- Evaluate expressions at specific values of their variables
- Evaluate formulas

Additional Assessments:

Constructed response:

Rectangle Perimeter (Writing expressions. One question that also has teacher commentary)

The Djinni Offer

(Exponential growth and connects growth to expressions involving exponents. Has teacher commentary)

Informal assessment:

Vocabulary Concept Matching (use throughout the unit)

Order of Operations Exit Ticket (Exit Ticket – 3 questions)

Algebraic Expression Card Game (2 – 4 players – matching card game)

Selected response:

Translating Algebraic Expressions (Matching & Multiple Choice – 6 questions)

Algebraic Expressions Matching (Matching – 8 questions)

Purchased Resources	Purchased Online Resources	Suggested Manipulatives

McGraw Hill Grade 6: Chapter 5 Lessons 1-4, 7 **McGraw Hill Grade 6 Plus:** Chapter 3 Lessons 1-4, 7 **Holt Mathematics Course 1 (old):** Chapter 1 Lessons 3-4 Chapter 2 Lessons 1-3 **Holt Mathematics in Context Level 1:** Expressions & Formulas Section E **Hands-on Standards *Grades 5-6:*** Algebra Lessons 4-5 **Hands-on Standards *Algebra:*** Expressions and Equations Lessons 1-2 **Hands-on Standards *Grade 6:*** Expressions and Equations Lessons 1-2	http://connected.mcgraw-hill.com/connected/login.do **Teacher User ID**: ccsde0(enumber) **Password**: cobbmath1 **Student User ID**: ccsd(student ID) **Password**: cobbmath1 **General Login**: User: georgiamath1 PW: demo123	Algeblocks Algebra Tiles Color Tiles

Web Resources

Translating Expressions Key Words
6.EE Rectangle Perimeter 3
6.EE Watch out for Parentheses
6.EE The Djinni's Offer
6.EE Seven to the What?!?
6.EE,G Sierpinski's Carpet
6.EE Rectangle Perimeter 1
6.EE Distance to School
http://www.mathsisfun.com/algebra/like-terms.html
https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-manipulating-expressions/v/combining-like-terms
https://learnzillion.com/lesson_plans/7583-simplify-algebraic-expressions-by-combining-like-terms#fndtn-lesson
http://mccluremathresources.weebly.com/math-6-resources.html

Vocabulary

- **Algebraic expression**: A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
- **Associative Property of Addition**: The sum of a set of numbers is the same no matter how the numbers are grouped.
- **Associative Property of Multiplication:** The product of a set of numbers is the same no matter how the numbers are grouped.
- **Coefficient:** A number multiplied by a variable in an algebraic expression.
- **Commutative Property of Addition:** The sum of a group of numbers is the same regardless of the order in which the numbers are arranged
- **Commutative Property of Multiplication:** The product of a group of numbers is the same regardless of the order in which the numbers are arranged.
- **Constant:** A quantity that does not change its value.
- **Exponent:** The number of times a number or expression (called base) is used as a factor of repeated multiplication. Also called the power.

- **Like Terms:** Terms in an algebraic expression that have the same variable raised to the same power. Only the coefficients of like terms are different.
- **Order of Operations:** The rules to be followed when simplifying expressions.
- **Term:** A number, a variable, or a product of numbers and variables.
- **Variable:** A letter or symbol used to represent a number or quantities that vary

Task Descriptions

Scaffolding Task	Task that build up to the learning task.
Constructing Task	Task in which students are constructing understanding through deep/rich contextualized problem solving
Practice Task	Task that provide students opportunities to practice skills and concepts.
Culminating Task	Task designed to require students to use several concepts learned during the unit to answer a new or unique situation.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications.
3-Act Task	Whole-group mathematical task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

State Tasks

Task Name	Task Type/ Grouping	Content Addressed	Standards
The Best Offer **(Spotlight Task)**	Constructing Task *Individual/Partner Task*	Exponents	MGSE6.EE.1
Exponents	Formative Task *Individual/Partner Task*	Exponents	MGSE6.EE.1
****Rules for Exponents**	Formative Task *Individual/Partner Task*	Exponent Conventions	MGSE6.EE.1
****Conjectures About Properties**	Formative Task *Partner/Small Group*	Properties of Numbers	MGSE6.EE.3 MGSE6.NS.4
Visual Patterns **(Spotlight Task)**	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
Perimeter and Area Expressions **(Spotlight Task)**	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
****The Algebra of Magic Part 1**	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b

			MGSE6.EE.2c
The Algebra of Magic Part 2	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
The Algebra of Magic Part 3	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
****Writing Expressions**	Formative Task *Individual/Partner Task*	Writing expressions	MGSE6.EE.2a
Writing and Evaluating Expressions	Formative Task *Group Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c

Topic 2: Equivalent Expressions

Big Ideas/Enduring Understandings:

Properties of numbers can be used to simplify and evaluate expressions.
Algebraic properties can be used to create equivalent expressions.
Two equivalent expressions form an equation.

Essential Questions:

- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How is the Distributive Property used to evaluate, simplify and expand expressions?
- How can I tell if two expressions are equivalent?

Student Relevance:

- Shortening lists and equivalency
- Shortening formulas

Content Standards

Content standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

MGSE6.EE.3 (820Q) Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

MGSE6.EE.4 (820Q) Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them.) *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

MGSE6.NS.4 (690Q) Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*

b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

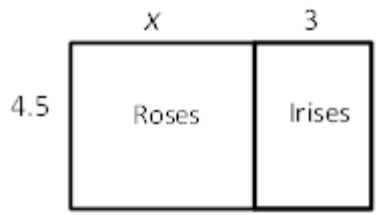
Vertical Alignment

5th Grade Standards	Math 7 Standards
MGSE5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them	**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

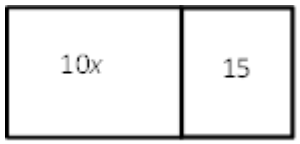
Instructional Strategies

EE.3

Students use the distributive property to write equivalent expressions. For example, area models from elementary can be used to illustrate the distributive property with variables. Given that the width is 4.5 units and the length can be represented by $x + 3$, the area of the flowers below can be expressed as $4.5(x + 3)$ or $4.5x + 13.5$.

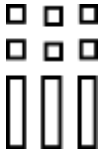


When given an expression representing area, students need to find the factors. For example, the expression 10x +15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length ($2x + 3$). The factors (dimensions) of this figure would be $5(2x + 3)$.



Students use their understanding of multiplication to interpret *3 (2 + x). For example, 3 groups of (2 + x).* They use a model to represent x, and make an array to show the meaning of *3(2 + x).* They can explain why it makes sense that *3(2 + x)* is equal to *6 + 3x.*

An array with 3 columns and *x* + 2 in each column:



Students interpret *y* as referring to one *y*. Thus, they can reason that one *y* plus one *y* plus one *y* ***must be*** *3y*. They also the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$: $y + y + y = y \times 1 + y \times 1 = y \times (1 + 1 + 1) = y \times 3 = 3y$

Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, $3x + 4x$ are like terms and can be combined as $7x$; however, $3x + 4x^2$ are not.

This concept can be illustrated by substituting in a value for *x*. For example, $9x - 3x = 6x$ not 6. Choosing a value for *x*, such as 2, can prove non-equivalence.

9(2) – 3(2) = 6(2)	**however**	9(2) – 3(2) ? 6
18 – 6 = 12		18– 6 ? 6
12 = 12		12 ≠ 6

Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Are the expressions equivalent? How do you know?

4m + 8 *4(m+2)* *3m + 8 + m* *2 + 2m + m + 6 + m*

Solution:

Expression	Simplifying the Expression	Explanation
4m + 8	*4m + 8*	Already in simplest form
4(m+2)	*4(m+2)* *4m + 8*	*Distributive property*
3m + 8 + m	*3m + 8 + m* *3m + m + 8* *(3m + m) + 8* *4m + 8*	*Combined like terms*

2 + 2m + m + 6 + m	*2 + 2m + m + 6 + m* *2 + 6 + 2m + m + m* *(2 + 6) + (2m + m + m)* *8 + 4m* *4m + 8*	*Combined like terms*

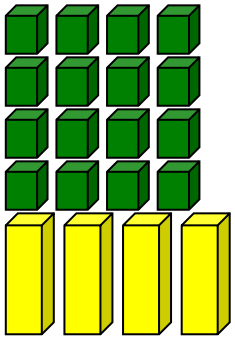
- It is expected that students will have prior knowledge/experience related to using parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols, writing and interpreting numerical expressions, generating two numerical patterns using two given rules, interpreting a fraction as division, and operations with whole numbers, fractions, and decimals. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these concepts.
- Sample Lesson Plans
  - Number Properties (PowerPoint)
  - Simplifying Expressions (PowerPoint)

Opening

- Vocabulary Review Game - Password
- Study Jams - Distributive Property
- **Bubble Man** - On a post-it note, students will draw a stick person with a dialogue bubble for "before" and one for "after". They will fill out the "before" bubble with what they already know about expressions. At the conclusion of the lesson, students fill in the "after" bubble. The teacher may post their responses on a poster.

Work Session

- Students use their understanding of multiplication to interpret *4(3 + x). For example, 4 groups of (3 + x).* They use a model to represent x, and make an array to show the meaning of *4(3 + x).* They can explain why it makes sense that *4(3 + x)* is equal to *12 + 4x.* Example: An array with 4 columns and *x* + 3 in each column:



- Students interpret *x* as referring to "one *x.*" Thus, they can reason that one *x* plus one x plus one *x* plus one *x* ***must be*** *4x.* Students will use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that *x* + *x* + x+ x = 4*x*:

x + *x* + x+ x = x ● 1 + *x* ● 1 + *x* ● 1 + *x* ● 1 = x ● (1 + 1 + 1+ 1) = *x* ● 4 = 4*x*

- Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example: Are the expressions equivalent? How do you know?

3z+ 6 *3(z+2)* *2z + 6 + z* *1 + z + z + 5 + z*

Solution:

Expression	**Simplifying the Expression**	**Explanation**
3z + 6	*3z + 6*	Simplest form
3(z+2)	*3(z+2)* *3z + 6*	*Distributive property*
2z + 6 + z	*2z + 6 + z* *2z + z + 6* *(2z + z) + 6* *3z + 6*	*Combined like terms*
1 + z + z + 5 + z	*1 + z + z + 5 + z* *1 + 5 + z + z + z* *(1 + 5) + (z + z + z)* *6 + 3z* *3z + 6*	*Combined like terms*

- Equivalent Expressions (answers on p. 2) – Students could complete the activity in groups of three where each student in the group is responsible for one problem in the row of 3. While the one student is answering the questions, the other two could serve as coaches or peer tutors. All must agree on the final solution.
- Equivalent Expressions "Balance Tilt" – Students could complete the activity in pairs, and they should write their answers in complete sentences.
- Properties Review Puzzle
- **Performance Task**: Conjectures About Properties
- **Performance Task**: Are We Equal?

Closing

- Unit 3 KIM Chart – Students could complete components of the chart throughout the unit as a review at the end of each lesson that addresses the concepts.
- **5 Words 3 Words** - As a class, small group, or individually, generate a list of 5 words that come to mind when thinking about equivalent expressions, then narrow it down to three most important words, and finally, the class picks one word that sums up their learning.
- **One Question I still have…** - On a post-it, students will write one question they still have about equivalent expressions. The teacher addresses the questions during the opening of the next class period.

Engage NY Lessons:
Grade 6 Module 4: Expressions and Equations

Topic D: Expanding, Factoring, and Distributing Expressions

- Lesson 9: Writing Addition and Subtraction Expressions
- Lesson 10: Writing and Expanding Multiplication Expressions
- Lesson 11: Factoring Expressions
- Lesson 12: Distributing Expressions
- Lessons 13–14: Writing Division Expressions

Differentiation

Make use of algeblocks for students that struggle and algebra tiles for students that do not.

Common Misconceptions

Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like x^3, 4x, 3(x + 2y) is critical. The fact that x^3 means (x)(x)(x) which is x times x times x, not 3x or 3 times x; 4x means 4 times x or x + x + x + x, not forty-something.

Evidence of Learning

By completion of this lesson, students should be able to:

- Apply properties of operations to generate equivalent expressions
- Identify when two expressions are equivalent

Additional Assessments

Constructed response:

Equivalent Expressions (12 True & False; if false students must make it true; answer key on 2nd page)
Equivalent Expressions "Balance Tilt" (Solve equation and justify answer)
Distance to School (Equivalent Expression – One question w/teacher commentary)
Equivalent Expression (One question, determine which expressions are equivalent otherwise create an equivalent expression. Teacher commentary included)
Rectangle Perimeter 2 (This is a follow-up to Rectangle Perimeter in lesson 1.)
Triangular Tables (One question with 4 parts. Teacher commentary included)

Informal assessment:

Password (Vocabulary PowerPoint Game)
Properties Review Puzzle (Holt worksheet with answer key)
Simplifying Expressions (Holt worksheet with answer key)

Purchased Resources	Purchased Online Resources	Suggested Manipulatives
McGraw-Hill Georgia Math Grade 6: Chapter 5 Lessons 5-7 **McGraw-Hill Georgia Math Grade 6 Plus:** Chapter 3 Lessons 5-7 **Holt Mathematics Course 2 Text (old):** Chapter 1 Lesson 6 **Hands-on Standards Grades 5-6:** Algebra Lessons 1-3 **Hands-on Standards Grades 7-8:** Algebra Lessons 8 **Hands-on Standards Algebra:** Expressions & Equations Lesson 3 **Hands-on Standards Grade 6:** Expressions & Equations Lessons 3-5 **Graphing Calculator Strategies Algebra:** Lessons 1-2	http://connected.mcgraw-hill.com/connected/login.do **Teacher User ID**: ccsde0(enumber) **Password**: cobbmath1 **Student User ID**: ccsd(student ID) **Password**: cobbmath1 **General Login**: User: georgiamath1 PW: demo123	Algebra Tiles or Alge-blocks (download printable Algebra Tiles here) From the National Library of Virtual Manipulatives: Online algebra tiles that can be used to represent expressions and equations. Online game **Late Delivery.** In this game, the student helps the mail carrier deliver five letters to houses with numbers such as $3(a + 2)$

Web Resources

6.EE,RP 7.EE,RP Anna in D.C.
http://hotmath.com/hotmath_help/topics/equivalent-expressions.html
https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-manipulating-expressions/v/equivalent-forms-of-expressions-1
https://learnzillion.com/lesson_plans/8904-read-and-write-equivalent-expressions-with-variables-and-exponents#fndtn-lesson
http://mccluremathresources.weebly.com/math-6-resources.html

Vocabulary

- **Distributive Property:** The sum of two addends multiplied by a number is the sum of the product of each addend and the number.

Task Descriptions

Scaffolding Task	Task that build up to the learning task.
Constructing Task	Task in which students are constructing understanding through deep/rich contextualized problem solving
Practice Task	Task that provide students opportunities to practice skills and concepts.
Culminating Task	Task designed to require students to use several concepts learned during the unit to answer a new or unique situation.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications.
3-Act Task	Whole-group mathematical task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

State Tasks

Task Name	Task Type/ Grouping	Content Addressed	Standards

Laws of Arithmetic	Formative Assessment Lesson *Individual/Partner Task*	Expressions	MGSE6.EE.3
****Are We Equal?**	Formative Task *Individual/Partner Task*	Identify equivalent expressions, Use properties to generate equivalent expressions	MGSE6.EE.3-4 **MGSE6.NS.4**
Culminating Task: **Sweet Tooth Chocolate Shop**	Summative Performance Task *Individual/Partner Task*	Expressions	MGSE6.EE.1-4