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ESP: EXTRAORDINARY SOLUTION PREDICTION

Topic
Solving equations

Key Question
How can you use algebraic symbols to determine how an ESP problem works?

Learning Goals
Students will:
- learn to translate a procedure of math operations into algebraic expressions,
- learn to simplify algebraic expressions using distribution and combining like terms, and
- work backwards with compound algebraic expressions to solve equations.

Guiding Documents
Project 2061 Benchmarks
- *Mathematical ideas can be represented concretely, graphically, and symbolically.*
- *Numbers and shapes—and operations on them—help to describe and predict things about the world around us.*
- *State the purpose of each step in a calculation.*

*NCTM Standards 2000**
- *Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems*
- *Understand and use properties of operations, such as the distributivity of multiplication over addition*
- *Represent the idea of a variable as an unknown quantity using a letter or a symbol*
- *Express mathematical relationships using equations*
- *Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships*
- *Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations*

Math
Number
  operations
  distributive property
Algebra
  finding patterns
  solving equations

Integrated Processes
Observing
Comparing and contrasting
Generalizing
Applying

Materials
ESP Intro animation (see *Management 2)*
Student sheets
Calculators, optional

Background Information
In this "mind reading" display, the spectators are asked to choose a number. Then they are asked to use that starting number in a series of simple computations. The mind reader's power is that the end can be predicted, regardless of what number the spectator starts with, or the starting number can be predicted when given the ending number.

In an ESP problem, the numbers calculated often disguise what is actually going on. Translating and simplifying the operations at an algebraic level removes the disguise. It is obvious that if you increase a number by 10 and then remove 10, the result is the starting number. No one is amazed by *"Add 10, subtract 10—bingo, you have the starting number!"* But this obvious manipulation can be hidden in several steps, e.g., add two, multiply by five, subtract 10. This gives you five times the original number.

As students work through math problems algebraically, they begin to recognize methods of disguising what is being done. As a result, they can develop some very good "mind reading" problems of their own.

Consider the following ESP problem included in the *Extension* of this investigation. Be aware that the instructions were written for the year 2007 and need to be modified for other years by increasing the addition in step E. The answers given are for a 12-year-old who has already had his birthday this year.

Directions	Number	Algebra
Pick your favorite number from 0 to 10.	7	n
Multiply this number by 2.	14	$2n$
Add 5 to the product.	19	$2n + 5$
Multiply the sum by 50.	950	$100n + 250$
If you already had your birthday this year, add 1757. If you haven't had it, add 1756.	2707	$100n + 2007$
Subtract the four-digit year you were born.	712	$100n + 12$

You should get a 3-digit number. The digit in the hundreds place is the favorite number you choose. The remaining numbers in the tens and ones place are your age.

It is obvious if you take away the year you were born from the present year, you get your age. The present year of 2007 clearly shows up in the algebraic form. But 2007 is lost in the numbers when 250 (5 · 50) is added to 1757 along with the 100 times the original number. Getting the original number to the hundreds place is the result of multiplying it by two and then by 50.

As students work through these problems algebraically, they will recognize the power of algebra for solving problems and clarifying patterns.

The solution for the second extension problem is shown here for a 14-year-old with 47¢ in her pocket.

Directions	Number	Algebra
Enter your age in years.	14	a
Double your age.	28	$2a$
Add 5 to the product.	33	$2a + 5$
Multiply the sum by 50.	1650	$100a + 250$
Add the value, in cents, of the change in your pocket.	1697	$100a + 250 + c$
Subtract 365 from the sum.	1332	$100a - 115 + c$
Add 115 to the difference.	1447	$100a + c$
Divide the sum by 100.	14.47	$\frac{a + c}{100}$

The number to the left of the decimal point is your age, and the number to the right of the decimal point is the value of your change in dollars.

Management

1. The teacher instruction video *Part Four: Inverse Operations* is on the accompanying DVD. It is also available at the following URL: www.aimsedu.org/media/books/. It provides the rationale and suggestions for using this activity successfully in your classroom.
2. The animation *ESP Intro* is on the accompanying DVD and at the URL listed above. Make preparations in your classroom so students can view the animation. If viewing through a computer, a projector enhances the experience.
3. The motivational aspect to this activity lies in the mysterious nature of the presentation. Before beginning the lesson, you should be familiar with the solution equation for each problem and practice what mind-reading flourishes are to be included in the presentation.
4. If students master the algebraic modeling of the problem and working it backwards to solve the equation for the unknown starting number, make sure to have them work on the problems in the *Extension*. This will broaden their understanding of algebraic manipulation as well as let them see the power of using algebraic symbols to solve problems.
5. You may want to allow students to use calculators, especially on the *Extension* problems.

Procedure

1. If available, show the students the *ESP Intro* animation.
2. Set the stage for the activity by verbally listing the procedure of the first problem while students calculate their ending numbers. Ask a student to give you his or her ending number and practice your "ESP" by giving the starting number (to do this, subtract five from the ending number). Repeat this process with several students.
3. Distribute the first two student pages and have students compute and record the numeric outcome of each step of the procedure for the first problem.
4. Have students develop the algebraic expression for each step of the problem and encourage them to simplify at each step by using the distributive property and combining like terms.
5. Give the students several ending numbers and have them determine the starting numbers. If students have difficulty at this step, have them consider working backwards numerically to get started on the solution.
6. Have the students share the ways they determined the starting number. Some may choose to do it numerically, but they should be encouraged to develop algebraic skills.
7. Work through the other three problems in a similar manner.
8. As a class, have students consider all four problems and discuss what methods were used to disguise the mathematics that was going on during the problem.
9. Distribute the third student sheet. Ask each student to develop a procedure of his/her own for disguising what is being done mathematically. Have them record their steps on the student sheet and check to see that it works numerically and algebraically.
10. Allow students to practice their problems on each other and then exchange them so that each student checks someone else's problem to see how it works algebraically.

Connecting Learning

1. When you have the final expression and the ending number, how do you determine the starting number? [undo what has been done to the expression to get it back to the starting variable, do the same thing numerically to the ending number]
2. What methods did the developer of the ESP problems use to disguise what was being done mathematically? [split the process into several steps, e.g., an increase by 18 could be done by adding three and multiplying by six]
3. In what situations can't a problem be solved by working backwards with numbers? [when the original number is added or subtracted from the process]
4. What method(s) did you use when you made your own ESP problem?
5. How did these methods compare to those that your classmates used?
6. Which method(s) do you think are the most effective? Why?

Extension

Provide students with the *ESP Extension* pages. Have them try the problems numerically to see what the problems involve. Then have the students try to develop some more interesting ESP problems of their own. *Extension Number One* needs to be adjusted every year. Before copying the page for students, write the correct numbers for each year in the three appropriate blanks.

Blank One	Blank Two	(Year Used) Blank Three
1757	1756	2007
1758	1757	2008
1759	1758	2009
1760	1759	2010
1761	1760	2011

Solutions

ESP One: $n + 5$
ESP Two: $2n + 1$
ESP Three: 10
ESP Four: n

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PREDICTIONS ONE & TWO

Extraordinary Solution Prediction One		
Directions	**Number**	**Algebraic Expression**
Choose a number.		
Add six to your number.		
Triple the sum.		
Decrease your product by three.		
Divide the difference by three.		

Extraordinary Solution Prediction Two		
Directions	**Number**	**Algebraic Expression**
Choose a number.		
Add six to your number.		
Multiply the sum by three.		
Add the original number.		
Add 12 to the sum.		
Divide the sum by two.		
Subtract 14.		

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PREDICTIONS THREE & FOUR

Extraordinary Solution Prediction Three

Directions	Number	Algebraic Expression
Choose a number.		
Add three to your number.		
Multiply the sum by four.		
Divide the product by two.		
Add 14.		
Take half the sum.		
Subtract the original number.		

Extraordinary Solution Prediction Four

Directions	Number	Algebraic Expression
Choose a number.		
Increase your number by five.		
Multiply the sum by five.		
Subtract 10 from the product.		
Divide the total by five.		
Subtract three.		

ESP: EXTRAORDINARY SOLUTION PREDICTION

YOUR PREDICTION

Your Extraordinary Solution Prediction

Directions	Number	Algebraic Expression

ESP: EXTRAORDINARY SOLUTION PREDICTION

EXTENSION NUMBER ONE

Extraordinary Solution Prediction Calculation

Directions	Number	Algebraic Expression
Pick your favorite number between zero and 10.		
Multiply this number by two.		
Add five to the product.		
Multiply the sum by 50.		
If you already had your birthday this year, add ______. If you haven't had it yet, add ______.		
Subtract the four-digit year you were born.		

You should have a three-digit number. The digit in the hundreds place is the favorite number you chose. The remaining numbers in the tens and ones places are your age.

Use algebra symbols and expressions to see how this problem works.

This set of directions only works for the year ________. How can you modify the directions to work for another year? Show your work.

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EXTENSION NUMBER TWO

Extraordinary Solution Prediction Calculation

Directions	Number	Algebraic Expression
Enter your age in years.		
Double your age.		
Add five to the product.		
Multiply the sum by 50.		
Add the value, in cents, of the change in your pocket. If you have more than a dollar, use only the last two digits of the number of cents. (For example, if you have $2.43, use 43.)		
Subtract 365 from the product.		
Add 115 to the difference.		
Divide the sum by 100.		

The number to the left of the decimal point is your age, and the number ot the right of the decimal point is the value of your change.

Use algebra symbols and expressions to see how this problem works. Try using two variables—one for age *(a)*, and one for cents *(c)*.

Connecting Learning

1. When you have the final expression and the ending number, how do you determine the starting number?
2. What methods did the developer of the ESP problems use to disguise what was being done mathematically?
3. In what situations can't a problem be solved by working backwards with numbers?
4. What method(s) did you use when you made your own ESP problem?
5. How did these methods compare to those that your classmates used?
6. Which method(s) do you think are the most effective? Why?

